PROBABILITY PROJECTS REVISITED

Maita Levine and Raymond H. Rolwing
University of Cincinnati
Cincinnati, Ohio

In an article entitled "'I Do and I Understand': A Project in Probability" in the November 1975 issue of The American Mathematical Monthly, we reported on our experiences incorporating projects into an elementary probability and statistics course. Since that time we have revised the assignment to include a computer simulation component. We would like to share the details of the new approach and the students' reactions to it.

The probability and statistics course which has incorporated the group project assignment meets four hours per week for two quarters. Since 1975, the average class size has increased to about 75 students and almost all of them are third or fourth year students in the College of Engineering. For most students it is a required course. For many of them it is an unwelcome requirement, since they fail to see its relevance to a career in engineering. Currently, the textbook is Probability and Statistics for Engineering and the Sciences by Jay L. Devore. During the first week of the term, the class is told that 30% of their final grade will be based on a probability project consisting of 15 problems. They are to work in groups of four, and they may choose their own group members. After a three week introduction to probability, the students are assigned to groups and the problems are distributed. Group assignments are made to insure that at least two members of each group are familiar with computer programming. Actually, experience indicates that most of the class have some programming skills and about half of the class have access to a microcomputer. The students are given six weeks to complete the assignment.

Generally, the problems are selected because they are classic examples of applications of probability, they are non-intuitive, they are suitable for computer simulation, or they are inherently interesting.

Among the problems assigned the past two years were the following:

1. A certain cook can prepare two cereals, Lumpies and Soggies, but sometimes she burns them. In fact, when she cooks Lumpies, her probability of burning it is 0.1. When she cooks Soggies, however, her probability of burning it is 0.4. Whenever she burns Lumpies, she cooks Soggies the next day. However, she really doesn't like Soggies very well, even when it isn't burned. Consequently, after cooking it one day, she always goes back to Lumpies. Begin with Lumpies. How often does the family eat Lumpies? How often does the family eat burned cereal?

2. Two players, 1 and 2, play the following game. Player 1 has a set of blue blocks and player 2 a set of red blocks. Player 1 starts by choosing at random one of the numbers 0 or 1. If she gets 0, she places a blue block at location 0; if she gets 1, she places a blue block at location 1. Player 2 repeats the procedure, then player 1, and so on. They continue placing their blocks on top of previously placed blocks, so that two towers are growing at locations 0 and 1. A player wins when, for the first time, the uppermost blocks on the two towers both show that player's color. What is the probability that the first player will win? What is the expected total number of blocks played?

3. Thirty-two ballots are cast, 20 for A, 12 for B. What is the probability that as the votes were counted, A always was leading?

4. Choose a number X at random between 0 and 1, and then draw a chord at distance X from the center of a circle of radius 1. What is the probability that the length of the chord will be less than the side of an inscribed equilateral triangle?

5. Consider a simple model for evolution. On a small island there is room for 2,000 members of a certain species. One year a favorable mutant appears. We assume that in each subsequent generation either the mutants take one place from the regular members of the species, with probability 0.6, or the reverse happens. Thus, for example, the mutation disappears in the very first generation with probability 0.4. What is the probability that the mutants eventually take over?

6. If a stick is broken at random into three pieces, what is the probability that the three pieces can be used to form a triangle?

The students are told that a few of the problems can be solved analytically by techniques already studied in class. In addition, a few of the problems represent applications of such topics as the negative binomial or hypergeometric distribution, not yet covered in class. The majority of the problems, however, require complex or creative counting techniques or the Markov process, which will not be discussed in class. Students are encouraged to use library resources, computer simulation, and of course, mathematical reasoning. They are to submit a complete or partial solution to each problem, including conjectures based on simulated results, references used, and a detailed description of their reasoning pattern.

Approximately one-third of the problems were solved using hints, solutions, or explanations found in textbooks or recreational mathematics books. Many students became acquainted with the mathematics library for the first time. And, many of them had their first experience with learning a mathematical topic independently. Linear algebra, matrix methods, and the programming languages associated with Texas Instruments or Hewlett-Packard calculators were most frequently learned with little help from the instructor. Computer or calculator simulations were used to produce solutions,or partial solutions,to about half the problems. It was somewhat disappointing to find that creative analytical reasoning was a relatively unpopular choice of methods.

At the end of the each quarter, the students were asked to evaluate the project and to suggest ways to improve it. Among their comments were the following:

> "The probability project was good...it allowed us the opportunity to try more challenging probability problems outside of class - problems that would stretch our minds a little more than the rather banal problems in the book. It was also a good opportunity to work with other people and bounce ideas off of them - to work as a group - to lessen the boredom."

"The group project was great! I learned a great deal about probability and simulating problems on a computer. Working with other people is always a great asset and making friends is always enjoyable. The assignment was a lot of fun."

"The probability project was a good lesson in real life. Too often we are expected to memorize equations to solve problems for a one hour test. The world is not like that. Given a problem, an engineer has the time and references to determine what to do. Computer simulation of some problems enormously simplified them... The problems were difficult and forced me to think... Doing a group project is a big help because everyone benefits from everyone else's insight."

"The project was a nice change from the usual homework assignment... It allows independent analysis...research...skills learned in class...creativity...use of our computers... The problems were fun."

In mathematics and engineering courses students are accustomed to encountering a lecture-discussion teaching technique and a set of rigidly controlled assignments. Furthermore, their undergraduate experiences tend to increase their already well developed competitiveness. Students' reactions to the probability projects, however, indicate that they welcome a less structured assignment and an opportunity to cooperate on a group effort. A course in probability and statistics is an ideal vehicle for providing a non-routine learning experience.

REFERENCES

H.D. Brunk, An Introduction to Mathematical Statistics, Ginn/Blaisdell, Waltham, Mass., 2nd ed., 1965.

J.G. Kemeny, J.L. Snell, and G.L. Thompson, Introduction to Finite Mathematics, Prentice-Hall, Englewood Cliffs, N.J., 3rd ed., 1974.

M.F. Levine and R.H. Rolwing, "'I Do and I Understand': A Project in Probability," The American Mathematical Monthly, November 1975.

F. Mosteller, Fifty Challenging Problems in Probability with Solutions, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1965.

L. Rade, Take a Chance With Your Calculator, Dilithium Press, Forest Grove, Oregon, 1977.

School Mathematics Study Group, Introduction to Probability, Part II, A.C. Vroman, Inc., Pasadena, Calif., 1966.

E. Woodward and J. Ridenhour, "An Interesting Probability problem," The Mathematics Teacher, December 1982.

ISOSCELES TRIANGLE PATTERNS

David R. Duncan and Bonnie H. Litwiller
University of Northern Iowa
Cedar Falls, Iowa

Teachers of mathematics at all levels of instruction are interested in practice activities which can be used by their students to develop skills or to maintain skills previously learned. Since mathematics is, in part, a study of patterns, many teachers are interested in activities which involve their students in conjecturing, investigating, and verifying. It is a serendipitous occurrence when activities can be found which at the same time lend themselves to the maintenance of computational skills and the discovery of patterns.

An example of this type of pattern involves drawing geometric designs on familiar number arrangements. Table 1 displays a sequence of nested isosceles triangles drawn on the interior of an extended addition table. We shall perform two activities on this sequence. (Continued on the next page)